

TM3 Problem 3-23

For a LRC circuit with $L = 0.1$ H, $C = 10 \mu\text{F}$ and $R = 10 \Omega$, find the oscillation frequencies, damping, period and amplitude after two periods.

The charge on the capacitor and current through the circuit are given by

$$\begin{aligned} \text{In[1]}:= & \text{Charge}[t_]= Q_{\text{zero}} * \text{Exp}[-\beta * t] * \text{Cos}[\omega_s * t] \\ & \text{Current}[t_]= \text{Charge}'[t] \end{aligned}$$

$$\text{Out[1]}= e^{-t\beta} \text{Cos}[t \omega_s] Q_{\text{zero}}$$

$$\text{Out[2]}= -e^{-t\beta} \beta \text{Cos}[t \omega_s] Q_{\text{zero}} - e^{-t\beta} \text{Sin}[t \omega_s] Q_{\text{zero}} \omega_s$$

$$\begin{aligned} \text{In[323]}:= & \text{Inductance} = 0.1; \\ & \text{Resistance} = 10; \\ & \text{Capacitance} = 10 * 10^{-6}; \end{aligned}$$

$$\omega_N = \sqrt{\frac{1}{\text{Inductance} * \text{Capacitance}}}$$

$$\beta = \frac{\text{Resistance}}{2 * \text{Inductance}}$$

$$\omega_s = \sqrt{\omega_N^2 - \beta^2}$$

$$\text{period} = \frac{2 * \pi}{\omega_s}$$

$$\text{decrement} = 2 * \text{period}$$

$$Q_{\text{zero}} = 10;$$

$$\text{Charge}[\text{decrement}]$$

$$\text{Current}[\text{decrement}]$$

$$\text{Out[326]}= 1000.$$

$$\text{Out[327]}= 50.$$

$$\text{Out[328]}= 998.749$$

$$\text{Out[329]}= 0.00629105$$

$$\text{Out[330]}= 0.0125821$$

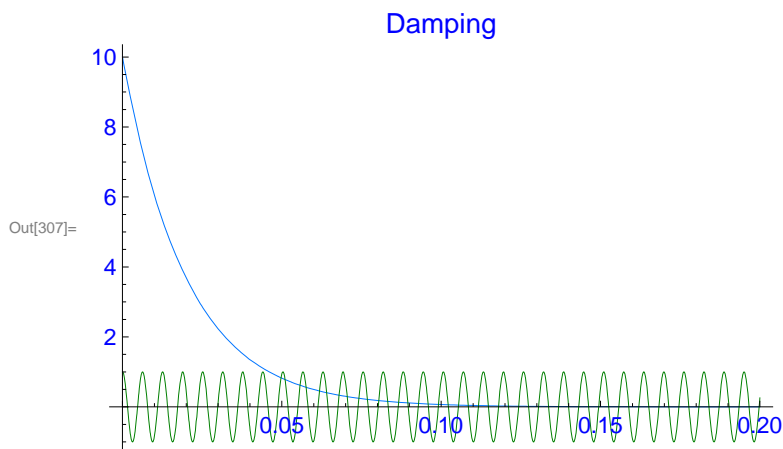
$$\text{Out[332]}= 5.33068$$

$$\text{Out[333]}= -266.534$$

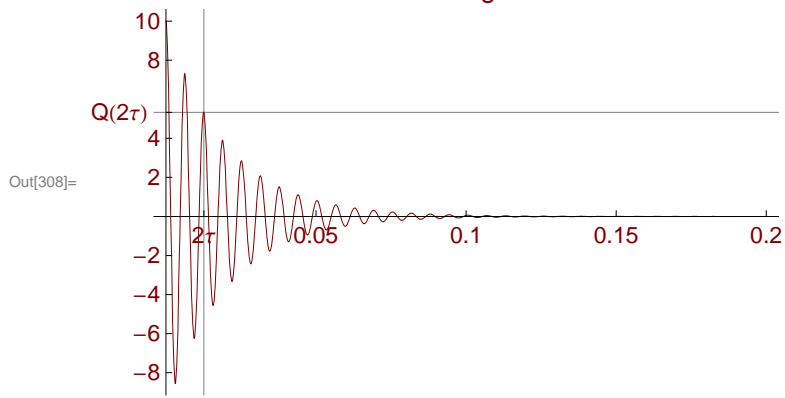
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In[305]:= pCos = Plot[Cos[ $\omega_s * t$ ], {t, 0, 0.2},
  BaseStyle -> {FontFamily -> Helvetica, FontSize -> 12, FontColor -> RGBColor[0, 0.5, 0]},
  PlotStyle -> RGBColor[0, 0.5, 0],
  PlotLabel -> "Oscillation"];
pAmp = Plot[ $Q_{zero} * \text{Exp}[-\beta * t]$ , {t, 0, 0.2}, PlotRange -> All,
  BaseStyle -> {FontFamily -> Helvetica, FontSize -> 12, FontColor -> RGBColor[0, 0, 1]},
  PlotStyle -> RGBColor[0, 0.46, 1],
  PlotLabel -> "Damping"];
Show[pAmp, pCos]
pCharge = Plot[Charge[t], {t, 0, 0.2}, PlotRange -> All,
  BaseStyle -> {FontFamily -> Helvetica, FontSize -> 12, FontColor -> RGBColor[0.5, 0, 0]},
  PlotStyle -> RGBColor[0.5, 0, 0], GridLines -> {{0.0126}, {5.33}},
  Ticks -> {{{0.0126, "2 $\tau$ "}, 0.050, 0.1, 0.15, 0.2},
    {-10, -8, -6, -4, -2, 0, 2, 4, {5.33, "Q(2 $\tau$ )"}, 8, 10}},
  PlotLabel -> "Charge"];
pCurrent = Plot[Current[t], {t, 0, 0.2}, PlotRange -> All,
  BaseStyle -> {FontFamily -> Helvetica, FontSize -> 12, FontColor -> RGBColor[1, 0.65, 0]},
  PlotStyle -> RGBColor[1, 0.65, 0], GridLines -> {{0.0126}, {-255.5}},
  Ticks -> {{{0.0126, "2 $\tau$ "}, 0.050, 0.1, 0.15, 0.2},
    {-8000, -6000, -4000, -2000, {-266.5, "I(2 $\tau$ )"}, 2000, 4000, 6000, 8000}},
  PlotLabel -> "Current"];
pChargeZoom = Plot[Charge[t], {t, 0, 0.05}, PlotRange -> All,
  BaseStyle -> {FontFamily -> Helvetica, FontSize -> 12, FontColor -> RGBColor[0.5, 0, 0]},
  PlotStyle -> RGBColor[0.5, 0, 0], GridLines -> {{0.0126}, {5.33}},
  Ticks -> {{{0.0126, "2 $\tau$ "}, 0.050, 0.1, 0.15, 0.2},
    {-10, -8, -6, -4, -2, 0, 2, 4, {5.33, "Q(2 $\tau$ )"}, 8, 10}},
  PlotLabel -> "Charge Zoomed"];
pCurrentZoom = Plot[Current[t], {t, 0, 0.05}, PlotRange -> {-500, 500},
  BaseStyle -> {FontFamily -> Helvetica, FontSize -> 12, FontColor -> RGBColor[1, 0.65, 0]},
  PlotStyle -> RGBColor[1, 0.65, 0], GridLines -> {{0.0126}, {-255.5}},
  Ticks -> {{{0.0126, "2 $\tau$ "}, 0.050, 0.1, 0.15, 0.2},
    {-8000, -6000, -4000, -2000, {-266.5, "I(2 $\tau$ )"}, 2000, 4000, 6000, 8000}},
  PlotLabel -> "Current Zoomed"];

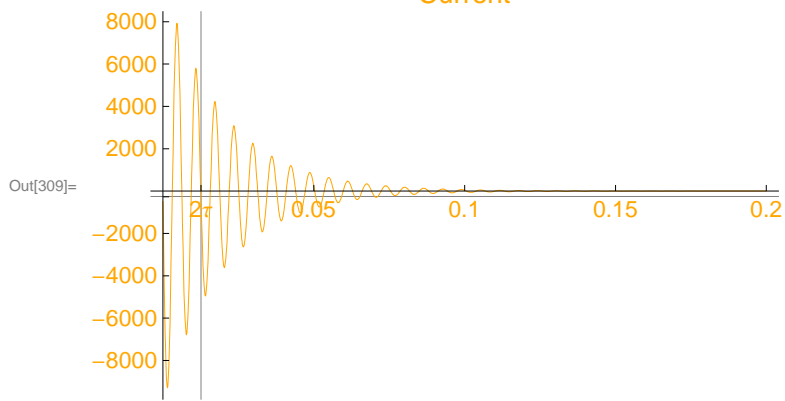
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Charge



Current



Charge Zoomed

